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INCREASING EVIDENCE indicates that algal levels in lakes are controlled by the external input of plant nutrients. Vollenweider[1] showed how the trophic states of a group of lakes could be related to the annual loading of phosphorus and the lake depths. Dillon[2] related phosphorus loading and flushing rate to trophic state in a group of Ontario lakes. Bachmann and Jones[3] demonstrated that a simple model related phosphorus inputs to algal levels in a group of lakes. The purpose of this paper is to develop a direct method of predicting summer levels of total phosphorus and chlorophyll *a* in a broad range of lakes. Such information should be of value to lake managers in making cost-benefit analyses of nutrient reduction programs to reduce algal densities.

METHODS

Measurements of total phosphorus and chlorophyll *a* concentrations were made on the surface waters of 16 Iowa lakes on several occasions in July and August 1974 (Table I). The methods are described elsewhere.[4] Phosphorus loading rates were estimated from the watershed areas of each of the lakes by using an average annual output of 0.35 kg/ha of P, which is typical for this region. Phosphorus loading from watersheds draining through lakes was estimated by using 0.065 kg/ha. Precipitation inputs on the lake surfaces were taken as 0.32 kg/ha.[5] Annual water inputs were calculated from the watershed areas and the annual runoff from the nearest USGS gauging stations. Lake evaporation was assumed to equal precipitation, and groundwater influences were ignored. Annual water inputs were divided by lake volumes to find the flushing rate for each lake.

Literature values were used to expand the sample to include 143 lakes, covering a broad range of trophic states; some lakes

TABLE I.—Original Data for 16 Iowa Lakes Sampled in July and August 1974

Lake	Surface Area (ha)	Mean Depth (m)	Phosphorus Loading (g/yr/sq m)	Flushing Rate (yr⁻¹)	July-Aug. Total P (mg/cu m)	July-Aug. Chlorophyll *a* (mg/cu m)
West Okoboji	1,540	11.9	0.17	0.05	36	6.8
East Okoboji	764	2.8	0.49	0.92	121	117.3
Big Spirit	2,168	5.2	0.26	0.21	43	29.1
Loon	291	1.5	0.57	2.56	85	73.7
Silver	432	1.8	0.54	1.19	150	262.2
Center	114	2.7	0.12	0.13	84	81.2
Lower Gar	98	1.1	1.13	6.25	117	194.2
Black Hawk	366	1.4	0.58	1.67	109	134.4
Storm	1,246	2.2	0.20	0.33	66	48.8
Lost Island	465	3.1	0.39	0.49	127	100.4
Trumbell	329	1.0	2.06	9.09	104	53.6
High	218	1.0	0.36	1.37	130	227.3
Ingham	144	1.5	0.60	0.86	88	63.4
North Twin	177	3.0	0.29	0.37	53	76.6
Cornelia	85	1.4	0.11	0.25	75	42.8
Clear	1,443	2.7	0.14	0.17	36	18.4

have data for several years.[3-32] All these lakes had measurements of summer total phosphorus and chlorophyll *a*. Fifty-one natural lakes had estimates of phosphorus loading rates, volume, and hydraulic flushing rates; four lakes had estimates for 2 yr. Large lakes, such as the Great Lakes, were excluded because they are incompletely mixed.

RESULTS

A strong correlation ($r = 0.95$) was found between the average July–August chlorophyll *a* concentrations and the measured concentrations of total phosphorus (Figure 1). This is in agreement with the previous findings.[10, 23, 32] The coefficients of our regression line are almost identical to those calculated by Dillon and Rigler.[10]

In view of the wide range of lake types and broad geographic distribution of lakes in the sample, it is surprising to find such a strong relationship between summer algal crops and a single nutrient element. A much greater scatter was expected because of (*a*) the effects of other limiting nutrients, (*b*) the effects of grazing on algal levels in different lakes, (*c*) the utilization

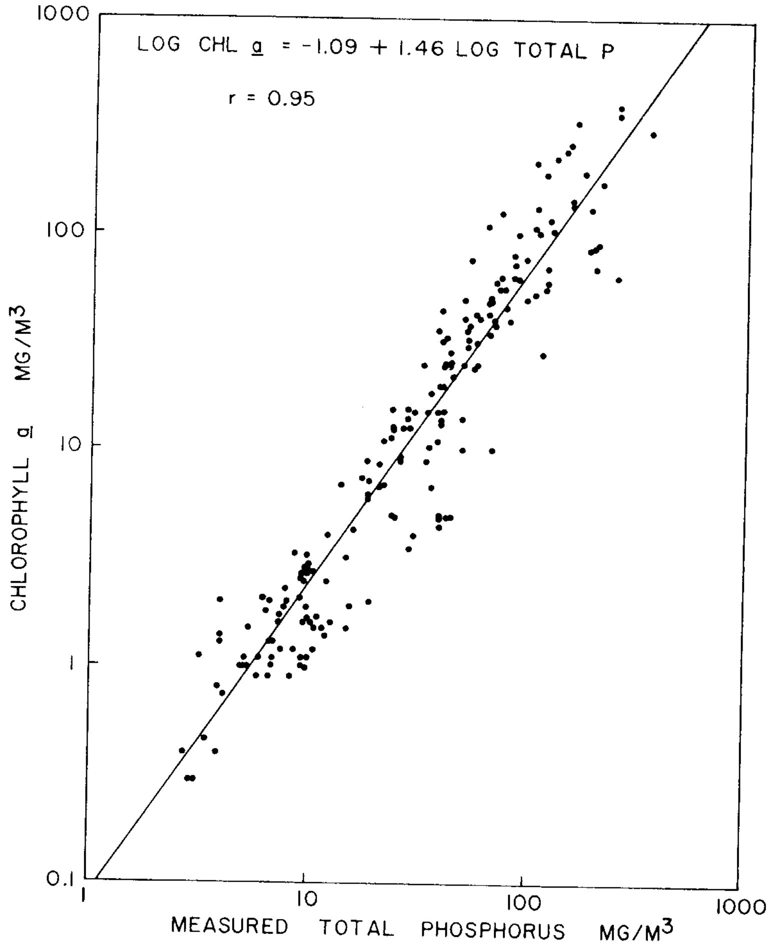

FIGURE 1.—Relationship between summer levels of chlorophyll *a* and measured total phosphorus concentration for 143 lakes.

of phosphorus by other parts of the food chain, and (*d*) the complexity of the phosphorus cycle and the fact that the total phosphorus measurement lumps together the many different phosphorus components found in the open waters of lakes.

The implication of this finding is that phosphorus is the element controlling algal biomass in a broad range of lakes. The slope of the log-log regression line is greater than one, indicating that the algal chlorophyll increases at a faster rate than does the phosphorus concentration. The ratio of chlorophyll *a* to phosphorus is greater at high concentrations of phosphorus than at low concentrations. This might be because of changes in the species composition of the algal populations in the richer lakes or because, at higher levels of phosphorus, a greater proportion of the total phosphorus is incorporated into the algal population.

The phosphorus-chlorophyll *a* relationship provides a means for understanding the differences in the algal densities of lakes and should provide a means of estimating the expected effects of changes in phosphorus levels on algal biomass in a given lake. This tool would be more useful if it were possible to predict the phosphorus concentration in a lake given basic information on phosphorus inputs, flushing rates, and basin morphometry.

Many models have been proposed to provide such information. These have been reviewed recently by Dillon.[33] The authors have chosen to follow the approach of Vollenweider,[34] who developed a differential equation based on the following assumptions:

1. The rate of supply of phosphorus, the flushing rate, and the sedimentation rate are constant through time.
2. The lake is considered as a continuously stirred reactor system (CSRS) defined as a single-compartment, open system.
3. The concentration of phosphorus in the outflow is the same as the concentration in the lake.
4. Sedimentation of phosphorus is proportional to the phosphorus concentration in the lake.

The steady-state solution is given by:

$$TP = \frac{L}{z(\sigma + p)} \qquad (1)$$

where:

TP = concentration of total phosphorus in the lake water, mg/cu m,
L = annual phosphorus loading per unit area of lake surface, mg/sq m,
z = mean depth of the lake, m,
σ = sedimentation rate, yr^{-1}, and
p = hydraulic flushing rate, yr^{-1}.

In developing this model, Vollenweider concluded that the concentration of phosphorus in lakes is determined largely by rate of supply, but is modified by losses through the outlet and to the sediments. The inclusion of the sedimentation term allows the model to be used for nonconservative substances, but potentially limits its usefulness because directly measuring sedimentation in the field requires more effort than would be involved in measuring the phosphorus and chlorophyll values rather than predicting them.

The sedimentation coefficient can be estimated from the data by rearranging the terms in Equation 1.

$$\sigma = \frac{L/z}{TP} - p \qquad (2)$$

Values ranged from −25.7 to 22.4, with a mean of 1.19 and a standard error of 0.69. Vollenweider [34] has listed values from 0.1 to 1.0. In this procedure, all the errors involved in estimating loading, mean depth, total phosphorus, and the flushing rate are incorporated into the sedimentation coefficient. Negative values might indicate a lake in which steady state does not exist and more phosphorus is being lost from the lake than is entering, or from the combined effects of the errors of estimation. No correlation could be found between the estimated sedimentation coefficients and the mean depth, loading rate, or total phosphorus concentration of the respective lakes. A slight positive correlation was found with the flushing rate.

The next approach was to see if a single sedimentation coefficient could be applied

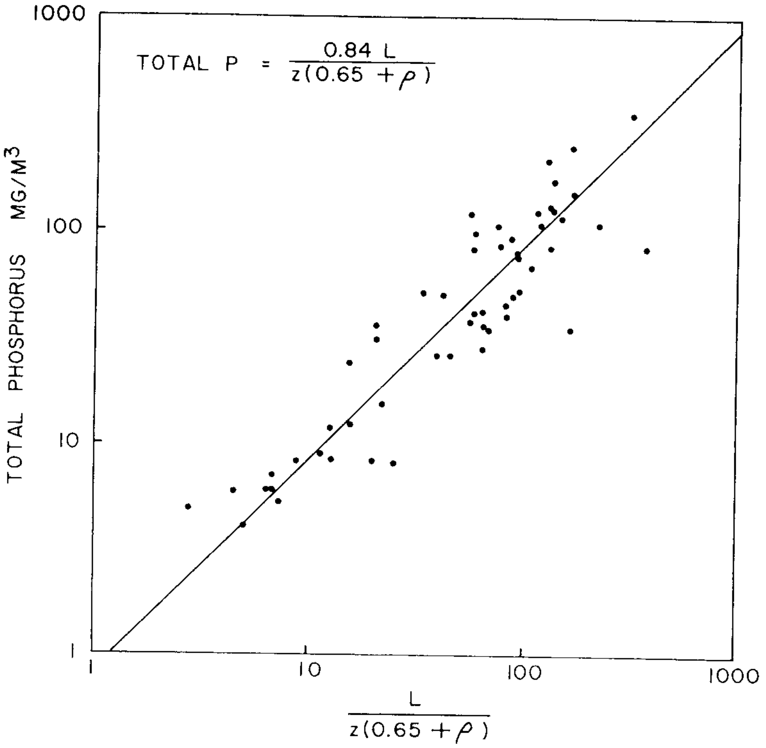

FIGURE 2.—Measured values of total phosphorus and calculated values based on Equation 1.

to all the lakes. A series of values in the range of 0.1 to 1.0 was tried. For each lake, the expression $L/z(\sigma + p)$ was calculated by using a trial value of σ; the correlation coefficient between that value and the measured total phosphorus also was determined. The largest correlation ($r = 0.918$) was for $\sigma = 0.65$. Correlation coefficients were above 0.913 for $0.4 \leq \sigma \leq 1.0$. In Figure 2, observed phosphorus values from the lakes in the data set are plotted against calculated values by using a σ value of 0.65 in Equation 1. Because the variance was believed to increase as the phosphorus concentrations increased, the slope constant of 0.84 was estimated by dividing the sum of the measured phosphorus concentrations by the sum of calculated phosphorus concentrations.[35] This indicated that, for this sample, actual phosphorus values were overestimated by approximately 16 percent. The best empirical equation for the prediction of total phosphorus concentrations is therefore:

$$TP = 0.84\ L/z(\sigma + \rho). \qquad (3)$$

An examination of the individual data points showed that most of the points with the highest deviation from the line were from a group of urban lakes in Minneapolis.[28] These lakes receive 30 to 70 percent of their annual inflow from storm drains and one lake received 80 percent of the annual inflow as groundwater. Stormwater is characteristically turbid and probably has sedimentation properties different from the runoff waters entering the other natural lakes in the sample. If these lakes are excluded from the analysis, the correlation coefficient increases to 0.94 rather than 0.92, and the slope constant increases from 0.84 to 0.97, indicating a near 1:1 relationship between observed and calculated values. Preliminary analyses on artificial reservoirs in Iowa indicate that sedimentation rates also are higher in these bodies of water, leading to overestimates in the predicted phosphorus concentrations. For this reason, the analyses were confined to natural lakes where good agreement is found.

This agreement does not necessarily prove the assumptions on which the Vollenweider model is based. Indeed, it is known that, for at least some of the lakes, the assumptions of constant rates of input and output are not met. Also, some of the lakes are stratified in the summer so that the entire water body is not being mixed during those times. Several other models were tried by using different assumptions, including those of Dillon[36, 37] and they gave a similar correlation to the one presented here. This model is an empirical equation that works reasonably well for predicting phosphorus concentrations over a broad range of lakes. Undoubtedly other equations can be formulated that will work about as well. The confidence bands are such that additional measurements would be necessary to produce more accurate predictions for an individual lake.

In Figure 3, the predicted phosphorus concentrations are plotted against the measured chlorophyll *a* concentrations. The two are highly correlated ($r = 0.89$). The regression line shown is the same one calculated for Figure 1. With few exceptions, the points fall along the same line, indicating that it should be possible to predict

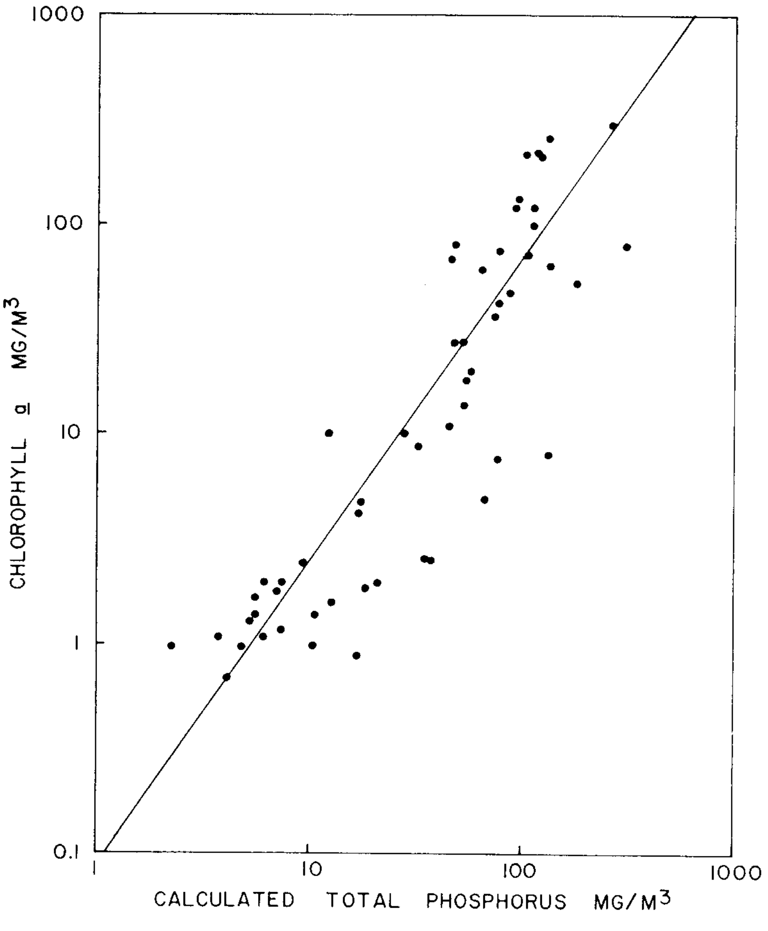

FIGURE 3.—Relationship between summer levels of chlorophyll *a* and calculated levels of total phosphorus.

both chlorophyll *a* and phosphorus concentrations for a lake from basic information on phosphorus inputs, lake volumes, and flushing rates. A reasonable estimate of annual phosphorus loading often can be made without an extensive limnological survey. Losses from specific soil types can be calculated with methods such as those outlined by Patalas[38] and Stewart and Markello.[39] Changes in the phosphorus input resulting from nutrient diversion or anticipated load increases could be estimated based on mean values from specific sources.[40]

The prediction equations developed here are surprisingly simple when one considers that they are based on a single nutrient, ignore various physical differences among lakes such as temperature, light penetrations, and stratification patterns, and treat the highly complex phosphorus cycle in such a simple manner. Yet they work quite well for the lakes in the sample. Undoubtedly, lakes will be found that do not fit the model because of local conditions. Even so, the equations provide a first approximation for the prediction of phosphorus and chlorophyll levels in lakes and should be of practical value.

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